International Journal of Engineering Science Invention
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org Volume 2 Issue 4 ǁ April. 2013 ǁ PP.16-22

Forecasting Exchange Rate between the Nigeria Naira and the US Dollar using Arima Models

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ABSTRACT: This paper describe an empirical study of modeling and forecasting time series data of Exchange rate of Nigeria Naira (₦) to the USD ($). The Box-Jenkins ARIMA methodology was used for forecasting the monthly data collected from January 1990 to December 2010. The diagnostic checking has shown that ARIMA (0,1,1) is appropriate. A four-year (48 months) forecast was made from January 2011 to December 2014, showing the Nigeria Naira in steady rate against the USD. These forecasts would be helpful for policy makers in both countries (Nigeria and the United State of America) to foresee ahead of time the Exchange rate, and the possible fluatation intervals of Naira to the USD for the period forecasted.
Key Words: Autocorrelation, Forecast, Dollar, ARIMA Model, Naira

I. INTRODUCTION

The Exchange rate reflects the ratio at which one currency can be exchange with another currency, namely the ratio of currency prices. It is the value of a foreign nation's currency in terms of the home nation's currency. It also specifies how much one currency is worth in terms of the other. A correct or appropriate exchange rate has been one of the most important factor for the economics growth in the economies of most developed countries, whereas a high volatility or inappropriate exchange rate has been a major obstacle to economic growth of many African countries of which Nigeria is inclusive. Volatility plays a very important role in any finacial market around the world and it has become an indispensable topics in financial markets for risk managers, portfolio managers, investors, academicians and almost all that have something to do with the financial markets (Richard, 2007). The consequences of substantial misalignments of exchange rates can lead to out contraction and extensive economic hardship. Moreover, there is reasonably strong evidence that the alignment of exchange rates has a critical influence on the rate of growth of per capital output low income countries (Isard, 2007). Therefore, forecasting accurately future volatility and correlations of financial asset returns is essential to derivative pricing, optimal asset allocation, fortfolio risk management, dynamic hedging and as an input for value-at-risk model. Forecasting is also a critical element of financial and managerial decision making (Majhi and Sahoo, 2009)

The Nigerian pound was introduced in 1959, and it external value was fixed at par with the British Pound Sterling which in turn defined its United States Dollar (USD) value as $2.80. Nigeria joined the International Monetary Fund (IMF) after independence, and the Nigeria Pound had its parity defined in June 1962 in terms of Gold at one Nigerian pound equals 2.48828 grams of fine gold. This confirmed its original USD per value. The naira replaced the Nigerian pound as Nigeria's currency in January 1973, its per value was set at half that of the pound. Hence the exchange rate became $1.52 to the naira. The rigid relationship between the USD and the Naira was terminated in April 1974; the fixed rate for sterling had been broken earlier in June 1972 when the sterling started to float officially. In February 1978, the system of determining the Naira exchange rate against a basket of currencies of Nigeria's main trading partners was finally adopted. However, as seen in Fig. 1, the value of the Naira against the USD has been non stationary. Hence, forecasting a variable in the financial markets is a matter of imperative importance, especially in a country like Nigeria.

II. MATERIALS AND METHODS

The formulation of ARIMA model depends on the characteristics of the series. In this paper, we have used the Exchange rate data of USD per Naira for the past 21 years (252 months) from January 1990 to December 2010. This data was taken from Central Bank of Nigeria Statistical Bulletin, 2010, retrievable from the website http://www.cenbank.org/. The R software, an open souce (GPL), interactive statistical environment modeled after S and S-plus was used in plotting the graphs and statistical analysis of the data set. The data were modeled using

Autoregressive Integrated Moving Average (ARIMA) stochastic model, popularized by Box and Jenkins (1976). An ARIMA (p,d,q) model is a combination of Autoregressive (AR) which shows that there is a relationship between present and past values, a random value and a Moving Average (MA) model which shows that the present value has something to do with the past residuals. The ARIMA process can be defined as;

$$\left(1 - \emptyset_1 L - \emptyset_2 L^2 - \cdots\cdots - \emptyset_p L^p\right) Y_t = (1 - \theta_1 L - \theta_2 L^2 - \cdots\cdots - \theta_q L^q) e_t \,. \tag{1}$$

Succintly as,

$$\emptyset(L)(1 - L)^d Y_t = \theta_q(L) e_t \,. \tag{2}$$

where,

Y_t = Represents the Exchange Rate of USD per Nigeria Naira

L = Represents the lag operator

$(1 - L)^d Y_t = \nabla^d Y_t$ is the series of the dth difference

$\emptyset_i$ = The ith autoregressive parameter

θ_i = The ith moving average parameter

e_t = The white noise

p, q and d denote the autoregressive, moving average and differenced order parameter of the process, respectively and ∇, the difference.

The estimation of the model consists of three steps, namely: identification, estimation of parameters and diagnostic checking.

Identification step: Identification step involves the use of the techniques to determine the values of p,q and d. The values are determined by using Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). This can be done by observing the graph of the data or autocorrelation and the partial autocorrelation functions (Makridakis et al, 1998). For any ARIMA (p, d, q) process, the theoretical PACF has non-zero partial autocorrelations at lags 1, 2, ..., p and has zero partial autocorrelations at all lags, while the theoretical ACF has non zero autocorrelation at lags 1, 2, …, q and zero autocorrelations at all lags. The nonzero lags of the sample PACF and ACF are tentatively accepted as the p and q parameters. For a non stationary series the log data is differenced to make the series stationary. The number of times the series is differenced determines the order of d. Thus, for a stationary data d = 0 and ARIMA (p, d, q) can be written as ARMA (p, q). However, this step has some difficulties, and involves a lot of subjectivity. It does on occasion happens that evidence examined at this stage may not point clearly in the direction of a single model (Salau, 1998). The best model for this study was selected based on the minimum value of Normalized Bayesian Information Criterion (BIC), Akaike Information Criterion (AIC) and Hannan-Quinn Criterion (HQ).

Estimation of parameters: This deal with estimation of the tentative ARIMA Model identified (selected) in the first step.

Diagnostic checking: This is concerned with checking the statistical significance of the model. The derived model must be checked for adequacy by considering the properties of the residuals whether the residuals from an ARIMA model is normally and randomly distributed. The Histograms and qq plots of the residuals can be used to assess the normality assumption visually. An overall check of the model adequacy is provided by Ljung-Box Q statistics. The test statistics Q is given in the equation below,

$$\mathrm{Q}_m = n(n+2) \sum_{k-i}^{n} (n-k)^{-1} r_k^2 \approx \chi^2_{m-r} \,. \tag{3}$$

where,

$\mathrm{r}_\mathrm{k}^2 = \text{The residual autocorrelation at lag k}$

$\mathrm{n} = \text{The number of residuals}$

$$\mathrm{m} = \text{The number of time lags included in the test}$$
$$\mathrm{Q} = \text{The modified Lung} - \text{Box test statistics}$$

If the p-value associated with the Q Statistics is small (p-value < α), the model is considered inadequate. The analysts should consider a new or modified model and continue the analysis until a satisfactory model has been determined.

III. RESULTS AND DISCUSSION

The Box-Jenkins's methodology for forecasting requires the series to be stationary. The data was found to be non-stationary, the log transformed "taken" before differencing to attain statinarity. An examination of Fig. 1, clearly revealed that non-stationarity is inherent in the data. Applying a transformation to address nonconstant variance is regarded as a "first step" (Tebbs, 2011). Then using a power transformation introduced by Box and cox, (1964). We observe that the log data on the chart seems stationary when the first-order difference was taken (see **Fig. 2).**

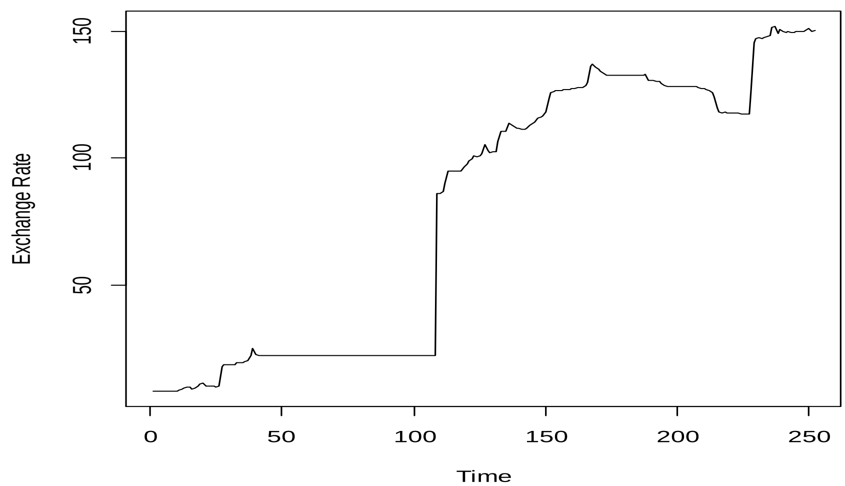


Fig. 1: Time Plot of Nigeria Naira to US Dollar Exchange Rate Jan. 1990-Dec. 2010

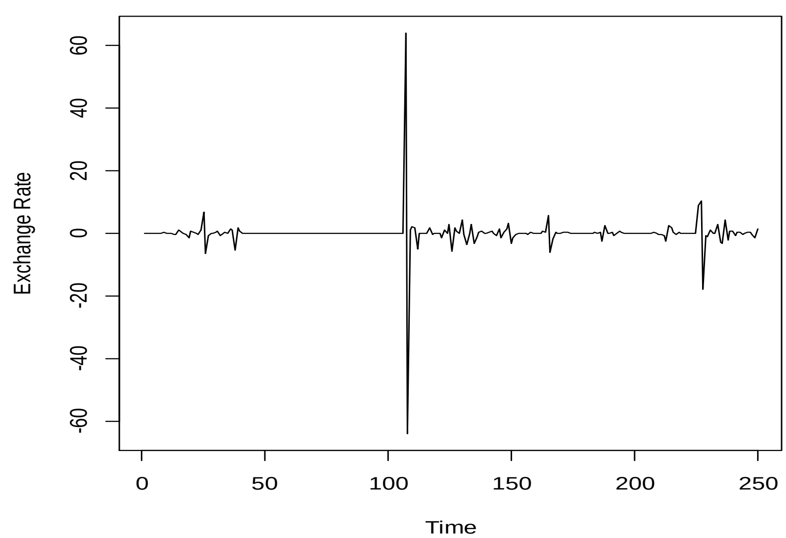


Fig. 2: The differenced log Exchange Rate of Naira to Dollar fron Jan. 1990-Dec.2010

The KPSS test and Augmented Dickey-Fuller test was used to verify whether or not the differenced series is stationary, and whether or not there is unit root. The sample ACF and PACF, shown in Fig. 3, confirm the tendency of $\nabla \log(Y_t)$ to behave as a first-order moving average process as the ACF has only a significant peak at lag zero and the PACF is tailing offf. This would suggest the exchange rate data follows an MA(1) process, or log exchange rate data follows an ARIMA(0, 1, 1) model. The final step of model fitting is model choice or model selection (Shumway, Stoffer, 2011). Among the three model selection criteria studied in the literature to judge the fitness of the model, ARIMA (0,1,1) has the least criteria values from the sets of model and therefore seems to provide the best satisfactory fit to the logged differenced of the Exchange rate series (see Table 1)

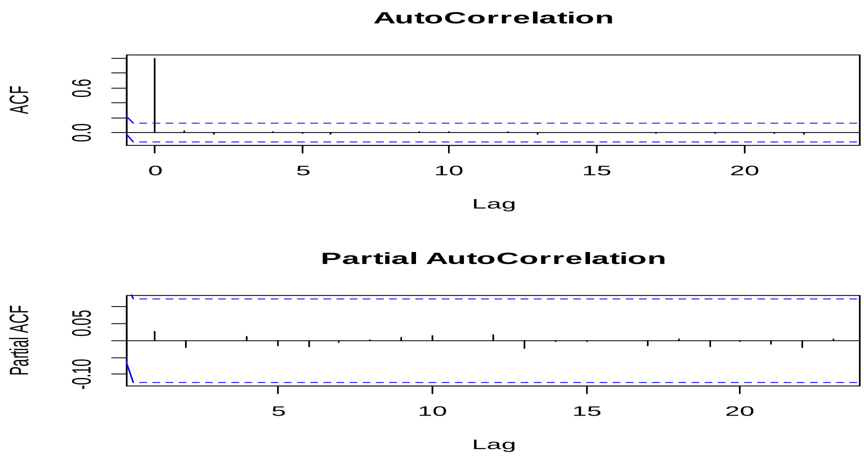


Fig. 3: Exchange Rate Correlogram of the differenced series (Jan. 1990-Dec.2010)

Table 1. Comparison of Selected ARIMA Models

ARIMA Model	Akaike Criterion(AIC)	Schwarz Criterion(BIC)	Hannan-Quin Criterion(HQ)
ARIMA(1, 1, 1)	**−460.18**	**−458.98**	**−463.90**
ARIMA(0, 1, 1)	**−462.19**	**−461.39**	**−464.70**
ARIMA(1, 0, 1)	**−455.02**	**−453.41**	**−459.97**
ARIMA(1, 1, 0)	**−462.18**	**−461.38**	**−464.66**
ARIMA(1, 2, 1)	**−457.14**	**−456.34**	**−459.62**
ARIMA(1, 2, 0)	**−360.22**	**−359.42**	**−362.70**

Source: Researcher's Calculation

Then,

Y_t = diff(log(Exchange Rate))

$$Y_t = Y_{t-1} + e_t - \theta e_{t-1}\ ,\ e_t \sim \text{WN}\ (0, \sigma^2)\ . \tag{4}$$

$$\Delta log Y_t = e_t - \theta e_{t-1},\ e_t \sim \text{WN}\ (0, \sigma^2)\ . \tag{5}$$

The final ARIMA(0,1,1) model is estimated by Maximum likelihood estimation (MLE) including estimation of the parameter θ. Therefore the fitted estimated model is,

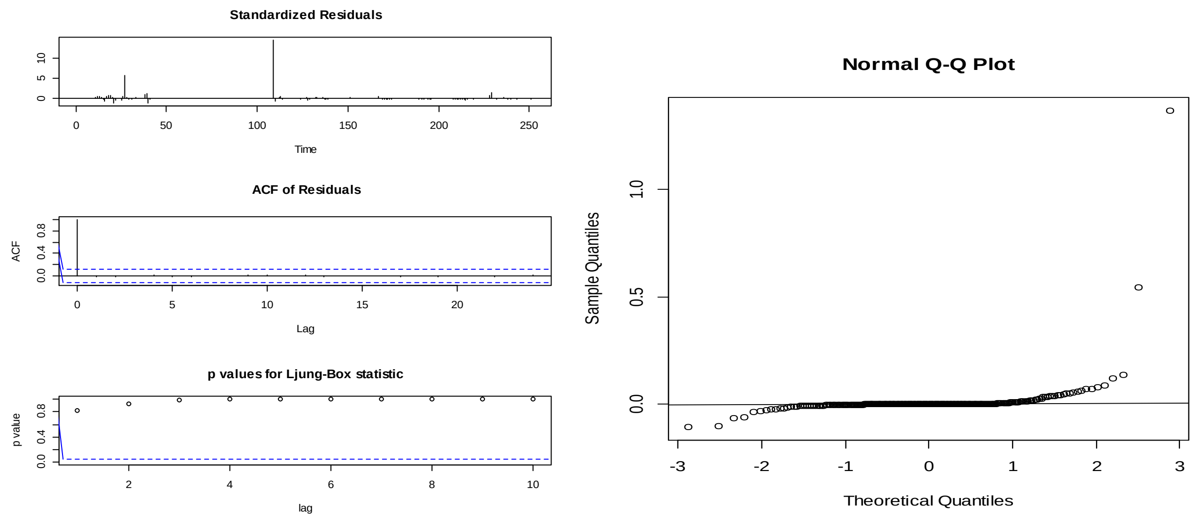
$$\Delta log Y_t = e_t + 0.042 e_{t-1}\ . \tag{6}$$

Equivalently,

$$log Y_t = log Y_{t-1} + e_t + 0.0429 e_{t-1}\ . \tag{7}$$

With estimate of the white noise variance ($\hat{\sigma}_e^2$) given as 0.009139 and s.e. = 0.0633

The Ljung Box-piece test for the model with lag = 10 has a p-value of 0.8224, thus having no evidence against ARIMA (0,1,1) model adequacy for the data. Again, examining the standard residual, autocorrelation residual, p-values for Ljung-Box statistic and the Normal Q-Q plot, as shown in Fig. 4, further lend support to ARIMA (0,1,1) model. Inspection of the time plot of the standardized residuals shows no obvious patterns. Notice that there are outliers, however, with a few values exceeding 10 standard deviations in magnitude. The ACF of the standardized residuals shows no apparent departure from the model assumptions, and the Q-statistic is never significant at the lags shown. The normal Q-Q plot of the residuals shows departure from normality at the tails due to the outliers that occurred. All this prove that the selected ARIMA model is an appropriate model.



(Fig. 4: Diagnostics of the residuals from arima(0,1,1) on diff (log (Exchange rate))

In time series modeling researchers are motivaed by the desire to produce a forecast with minimum errors as possible. The traditional Box-Jenkins approach is general and can handle effectively many series encounter in reality. Besides, previous research has demonstrated that the Box-Jenkins forecast out performs the Hot-Winters and stepwise autoregression forecasts (Newbold and Granger, 1974). The model was fitted for four years (48 months) period after the diagnostic test confirmed the model adequacy. Fig.5 shown the plot of the series containing the observe data, forecast and the interval (boundary) and Table 2 shows the values of **Fig. 5**

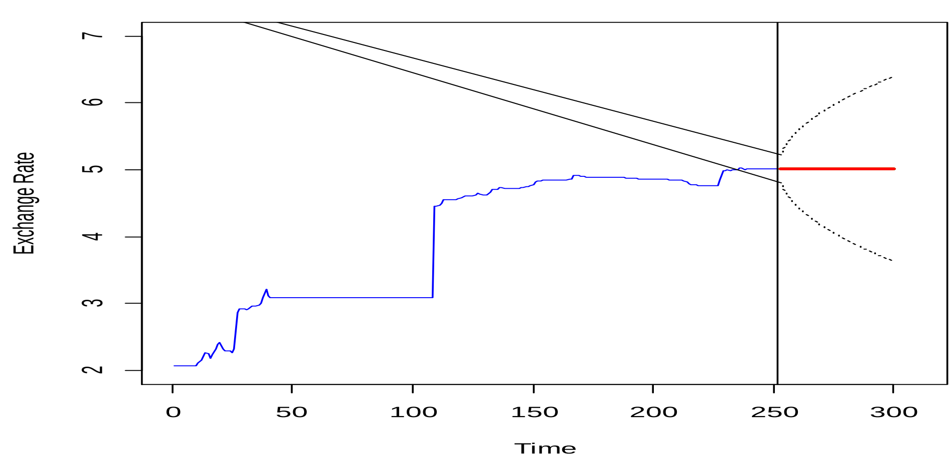


Fig. 5: Forecasts and Prediction intervals for Exchange Rate.
The vertical dotted line separates the log data from the predictions.

Table 2. Forecast Values (Jan. 2011-Dec.2014)

Forecasts from January 2011 to December 2014 (48 months) with 95% Confidence Limits		
Months/Year	**Prediction Intervals**	**Prediction**
Jan., 2011	(4.826692, 5.201432)	5.014062
Feb., 2011	(4.743342, 5.284782)	5.014062
Mar., 2011	(4.680189, 5.347935)	5.014062
April, 2011	(4.627212, 5.400913)	5.014062
May, 2011	(4.580663, 5.447462)	5.014062
June, 2011	(4.538649, 5.489475)	5.014062
July, 2011	(4.500059, 5.528065)	5.014062
Aug., 2011	(4.464170, 5.563954)	5.014062
Sept., 2011	(4.430484, 5.597640)	5.014062
Oct., 2011	(4.398639, 5.629485)	5.014062
Nov., 2011	(4.368363, 5.659761)	5.014062
Dec., 2011	(4.339444, 5.688680)	5.014062
Jan., 2012	(4.311715, 5.716409)	5.014062
Feb., 2012	(4.285040, 5.743084)	5.014062
Mar., 2012	(4.259307, 5.768817)	5.014062
April, 2012	(4.234423, 5.793701)	5.014062
May, 2012	(4.210309, 5.817815)	5.014062
June, 2012	(4.186898, 5.841226)	5.014062
July, 2012	(4.164131, 5.863993)	5.014062
Aug., 2012	(4.141959, 5.886166)	5.014062
Sept., 2012	(4.120336, 5.907788)	5.014062
Oct., 2012	(4.099224, 5.928900)	5.014062
Nov., 2012	(4.078589, 5.949536)	5.014062
Dec., 2012	(4.058399, 5.969726)	5.014062
Jan., 2013	(4.038627, 5.989498)	5.014062
Feb., 2013	(4.019247, 6.008877)	5.014062
Mar., 2013	(4.000238, 6.027886)	5.014062
April, 2013	(3.981580, 6.046545)	5.014062
May, 2013	(3.963252, 6.064873)	5.014062
June, 2013	(3.945238, 6.082886)	5.014062
July, 2013	(3.927523, 6.100601)	5.014062
Aug., 2013	(3.910093, 6.118032)	5.014062
Sept., 2013	(3.892933, 6.135191)	5.014062
Oct., 2013	(3.876032, 6.152092)	5.014062
Nov., 2013	(3.859379, 6.168746)	5.014062
Dec., 2013	(3.842962, 6.185163)	5.014062
Jan., 2014	(3.826772, 6.201352)	5.014062
Feb., 2014	(3.810800, 6.217324)	5.014062
Mar., 2014	(3.795037, 6.233087)	5.014062
April, 2014	(3.779476, 6.248649)	5.014062
May, 2014	(3.764108, 6.264017)	5.014062
June, 2014	(3.748927, 6.279198)	5.014062
July, 2014	(3.733926, 6.294199)	5.014062
Aug., 2014	(3.719098, 6.309026)	5.014062
Sept., 2014	(3.704439, 6.323686)	5.014062
Oct., 2014	(3.689941, 6.338183)	5.014062
Nov., 2014	(3.675601, 6.352523)	5.014062
Dec., 2014	(3.661413, 6.366711)	5.014062

Source: Researcher's Calculation

The minimum mean squared error (MMSE) forecast on the original scale (back-transformed) given by (Tebbs, 2011),

$$\hat{Y}_{t(l)} = \exp\left\{\hat{Z}_t(t) + \frac{1}{2}(\mathrm{var}\,[e_t(l)])\right\}. \tag{8}$$

Where,

$Var\ [e_t(l)]$ = Variance of the *l-step* ahead forecast

$e_t(l) = Z_{t+1} - \hat{Z}_t(l)$

l = the *lead* times

$\hat{Z}_t(l)$ = *l-step* ahead MMSE forecast on the log scale

IV. CONCLUSION

Since the exchange rate between the Naira (₦) and the USD ($) was found to be non-stationary, Fig. 5 and Table 2, clearly shows that the naira will not have major fluctuation (upward or downward trend) against the USD, with probable exchange rate of ₦150.51k (one hundred and fifty naira fifty-one kobo) per USD ($) in the next 48 months (Jan., 2012-Dec., 2014). This may help policy makers to conduct a suitable monetary policy which will in turn achieve its desired objectives and higher economic activity. This may also help the policy makers in extracting useful information about the economic and financial conditions.

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